An Experimental Characterization of the Non-linear Rheology of Rock

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ABSTRACT

A laboratory experimental program is underway to constrain rheological models for deformation of rock in the brittle-damage and moderate-strain regimes. The data will be used to assess the importance and consequences of nonlinearities in mechanical properties which affect the amplitude and spectral character of regional and teleseismic signals. Experiments are being performed to characterize the intrinsic loading path dependence of the deformation of rock subjected to various loading conditions. Cyclic loading tests in unconfined uniaxial compression, torsion, and confined compression are being conducted on a variety of lithologies under conditions expected in the moderate-strain and brittle-damage regimes of an underground explosion. The effect of ambient normal load, water saturation, and loading rate on hysteresis in deformation are being tested directly.

OBJECTIVES

In order to discriminate between underground nuclear explosions and other seismic sources, it is necessary to have confidence in source spectra derived from regional and teleseismic signals. There are four regimes pertinent to the wave propagation problem:

- a near-source, or hydrodynamic regime
- a brittle-damage regime, where rock failure dominates the rheology
- a moderate-strain regime where rock deformation is known to be nonlinear, inelastic, and hysteretic, yet permanent deformation does not occur.
- a far field regime where rock deformation is linear and traditional seismological techniques can be applied.

In this work, we concentrate on studying fundamental issues concerning the deformation of rock in the moderate-strain regime and its transition into the brittle-damage regime.

The objectives are to provide laboratory data for the constraint and development of realistic rock rheologies suitable for modeling wave propagation in the moderate-strain and brittle-damage regimes. The data will be used to assess the importance of nonlinearities on the amplitude and spectral character of regional and teleseismic signals. This information is needed to better understand the extent to which nonlinear material properties in the brittle-damage and moderate-strain regimes are important to regional discriminants and event detection.

In coordination with others, numerical simulations of wave propagation using the developed nonlinear rheologies will be performed to assess the importance of the nonlinearities for problems concerning discrimination, detection, and yield estimation of underground explosions. Comparisons of the simulation results with field data will be made where appropriate.

PRELIMINARY RESEARCH RESULTS

Background

As a result of previous laboratory studies, a general rheological model has been developed for the case of uniaxial stress perturbations in unconfined compression [see *Boitnott*, 1993, 1994]. The model and supporting data provide constraints on the loading path dependence of the deformation for arbitrary loading history, and have provided us with a foundation upon which to develop more general rheologic models.

In order to apply the developed rheology directly to problems of wave propagation from a seismic source, we need to extend the rheology to loading paths expected in the field. Four main goals have been identified. First, we need to extend the rheology to include a wide variety of loading conditions. In order to model shear wave propagation, we need laboratory data to constrain hysteresis in the shear modulus during shear loading. In addition, in order to model compressional wave propagation (including the effects of spherical wavefronts) we need to extend the uniaxial-stress rheology to loading paths which are transitional between uniaxial-strain and uniaxial-stress. Second, we need to characterize the effects of confining pressure on the rheological models in order to address questions concerning the effect of depth of burial on the seismic signature of an underground explosion. Third, we need to examine in more detail the behavior at reversals in loading direction since the details at the cusps in stress-strain relationship strongly influence the extent to which energy is shifted to higher frequencies. Fourth, we need to extend the stress regime of the rheologies to include the brittle-damage regime.

Developing a More General Rheology

In order to extend the rheology to loading conditions other than uniaxial stress, three types of experiments are being performed. First, unconfined uniaxial stress experiments are being performed during which both axial and radial strain is measured. These experiments are motivated by the need to identify simplifying principles which can be used to extend the uniaxial-stress rheology to the case of arbitrary triaxial stress perturbations. For example, we might hope to model triaxial deformation through the use of superposition arguments, defining a change in stress/strain state as a superposition of a series of hysteretic and non-hysteretic components.

An example of data from an experiment on Sierra White granite is shown in Figure 1. In Figure 1a we see the familiar cusped hysteresis loop between axial stress and axial strain for a cyclic axial load in unconfined compression. In Figure 1b, we plot the corresponding radial strain as a function of axial stress. Note that the radial strain exhibits little (if any) measurable hysteresis. Importantly, this observation rules out the possibility of decomposing an arbitrary deformation into a non-hysteretic dilation and hysteretic distortion, since the data indicates considerable hysteresis in the volumetric strain during perturbations in uniaxial stress.

The observed lack of hysteresis in the radial strain during unconfined compression may however indicate a simplifying principle which may lead to a more general rheology capable of handling the transition from uniaxial-strain to uniaxial-stress loading perturbations. This is of considerable importance in applying the rheological model to propagation of non-planar compressional waves, where the loading path is intermediate (and variable in space in time) between uniaxial-strain and uniaxial-stress. The data in Figure 1b suggests that the radial (Poisson) expansion is largely an elastic phenomena. It follows that the confining pressure required to maintain uniaxial-strain conditions should also be non-hysteretic with respect to axial load. If true, we might hope to develop a rheological model which involves the simple addition of a uni-valued (non-hysteretic) dependence of the axial-strain vs. axial-stress relationship on the hydrostatic component of the stress state.

The notion that the difference between uniaxial-strain and uniaxial-stress loading may be included in the rheology with the rather simple addition of a non-hysteretic confining pressure effect is consistent with some limited observations. In Figure 2 we compare experimental data on Berea sandstone from *Hilbert et al.* [1994] with a simple extension of a rheological model recently developed for the case of unconfined compression. Figure 2a shows a plot of axial strain versus axial stress for uniaxial-stress and uniaxial-strain loading paths on a sample of Berea sandstone. During the overall loading and unloading, small perturbations in axial load were performed to

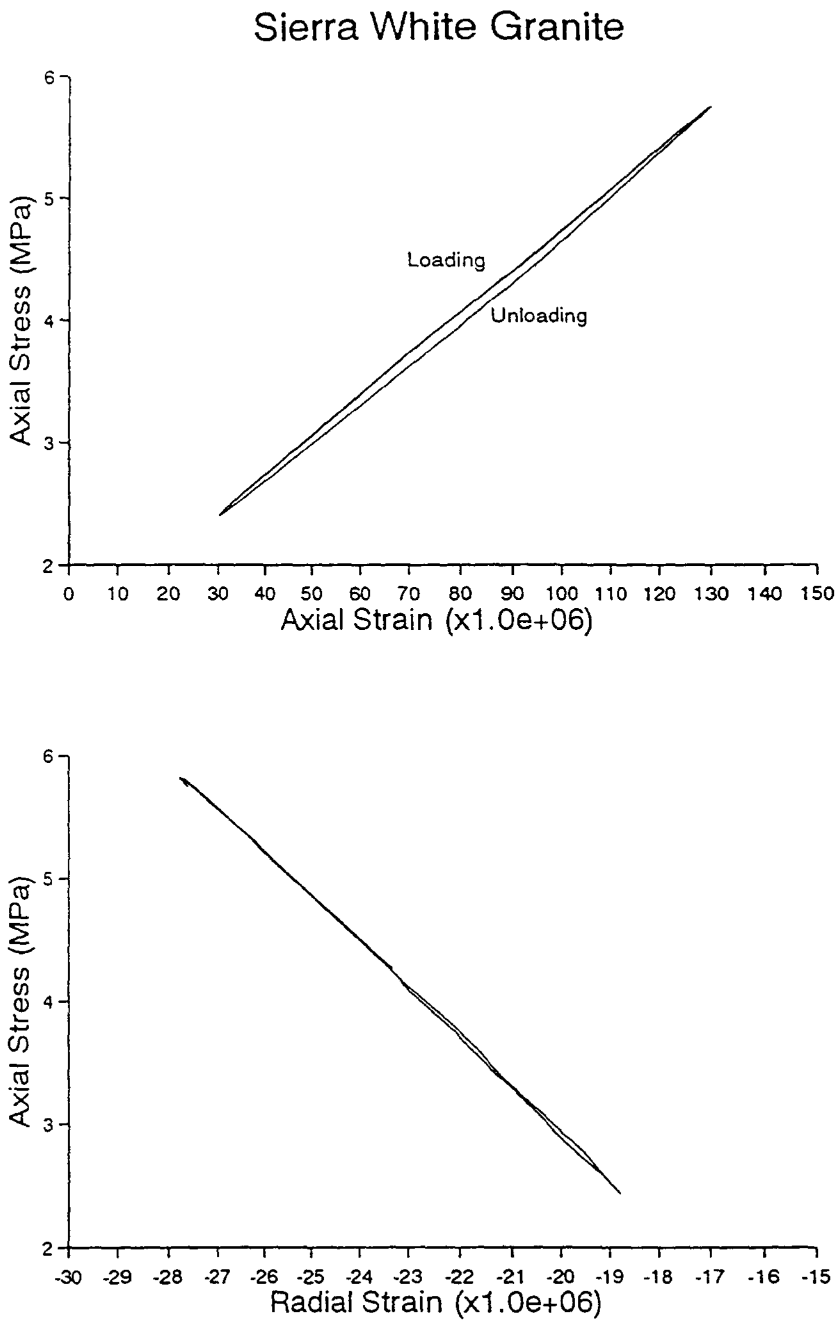
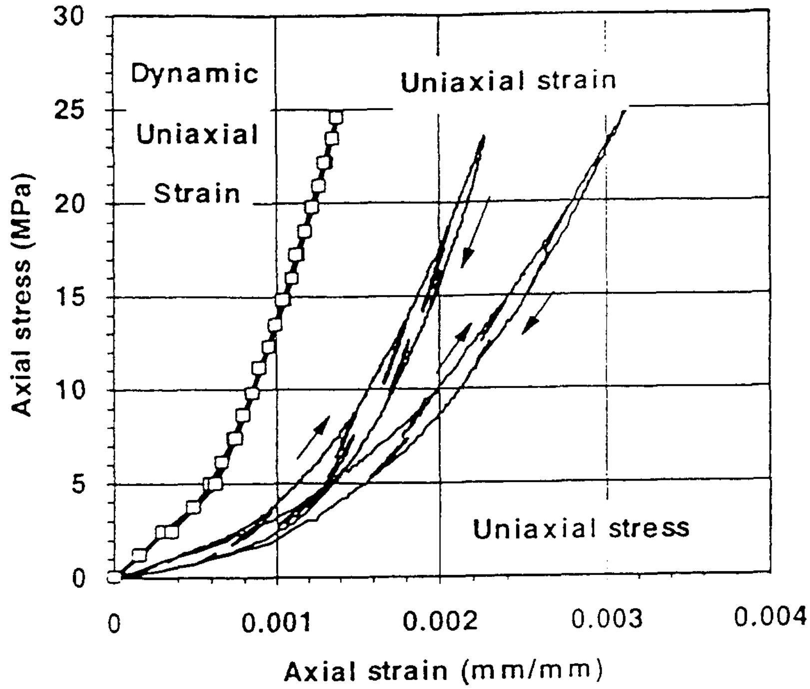


Figure 1: Axial stress versus axial strain and axial stress versus radial strain for cyclic loading of Sierra White granite in unconfined compression. Note the absence of hysteresis in the radial strain. This observation allows for the possibility of a simple model incorporating the effects of confining pressure on the hysteresis in axial strain.



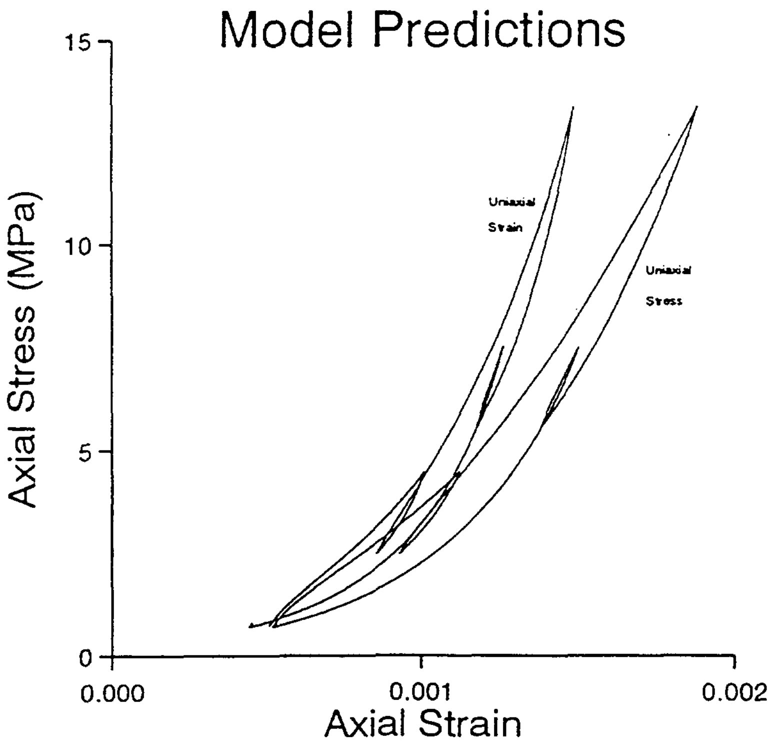


Figure 2: Comparison of Experimental data (from *Hilbert et al.*, 1994) with model predictions for hysteretic deformation in Berea Sandstone. A simple scaling law is used to incorporate the effects of changing confining pressure on the axial stiffness during uniaxial strain loading. The simple model appears to contain many of the features exhibited by the data. Model results are based on experiments using a different sample of Berea sandstone, and thus small quantitative discrepancies between model and data should not to be considered significant.

measure the response to small perturbations in load. Note that the character of the hysteresis is similar for the two loading conditions, and that the difference between the uniaxial-strain and uniaxial-stress behavior is largely the result of the increase in stiffness of the sample as the confining pressure is increased to enforce uniaxial strain conditions.

Model predictions based on the rheological model of *Boitnott* [1993] are shown in Figure 2b for a similar loading path. The peak stress for the model results is somewhat reduced from that of laboratory data since the parameterization of the model is only valid for stresses below 15 MPa. The model results for uniaxial-stress are simply the direct application of the model of Boitnott [1993] applied to the load protocol similar in form to that used by *Hilbert et al.* [1994]. The model results for uniaxial-strain are produced by adding a confining pressure effect to the the uniaxial-stress model. We start by defining the axial stiffness $\eta \equiv \partial\sigma_{11}/\partial\varepsilon_{11}$, where σ_{11} and ε_{11} are the axial stress and axial strain respectively. We then assume that $\eta \equiv \gamma E$, where E is the hysteretic Youngs modulus describing the rheology for uniaxial-stress perturbations, and γ a is scaling factor which is a linear function of the confining stress. The confining stress is assumed to change linearly with axial stress based on the observed lack of hysteresis in the radial-strain / axial-stress relationship as discussed in the context of Figure 1b. The resulting model provides a simple means through which to include confining pressure effects on the axial-stress/axial-strain relationship. Note that the model results compare favorably with the experimental data, illustrating that the scaled rheology is reasonable.

Guided by these observations, experiments are being conducted to constrain the confining pressure effect on hysteresis in uniaxial stress and comparing these results to axial-strain / axial-stress hysteresis during uniaxial-strain experiments. Experiments will be performed on a number of rock types. Once a rheology is developed, its predictive capabilities will be tested by performing a number of experiments using loading paths which mimic (in a quasi-static sense) the expected loading conditions for a spherically propagating compressional wave.

Torsion Experiments

In order to develop similar rheologies for shear wave propagation, torsional experiments are being conducted to constrain hysteresis in the shear modulus as a function of normal load and shear stress history. Experiments are being performed on hollow cylinders of intact sandstone and granite.

Examples of data on Berea sandstone are shown in Figure 3. In Figure 3a, the shear-stress vs. shear-strain relationship is plotted for a cyclic torsional load at a fixed axial load of 5.4 MPa. The frequency of the loading oscillation was 0.01 Hz and data from two different amplitude oscillations are shown. Note the nested nature of the hysteresis loops with amplitude and the fact that the mean slope of the hysteresis loops (i.e. the average shear modulus) decreases with loop amplitude. Figure 3b shows the results of three torsional oscillations at three different axial loads (3.6, 5.4 ,and 7.2 MPa). As expected, the average shear modulus increases, and the width of the hysteresis loops decreases, with increasing axial load.

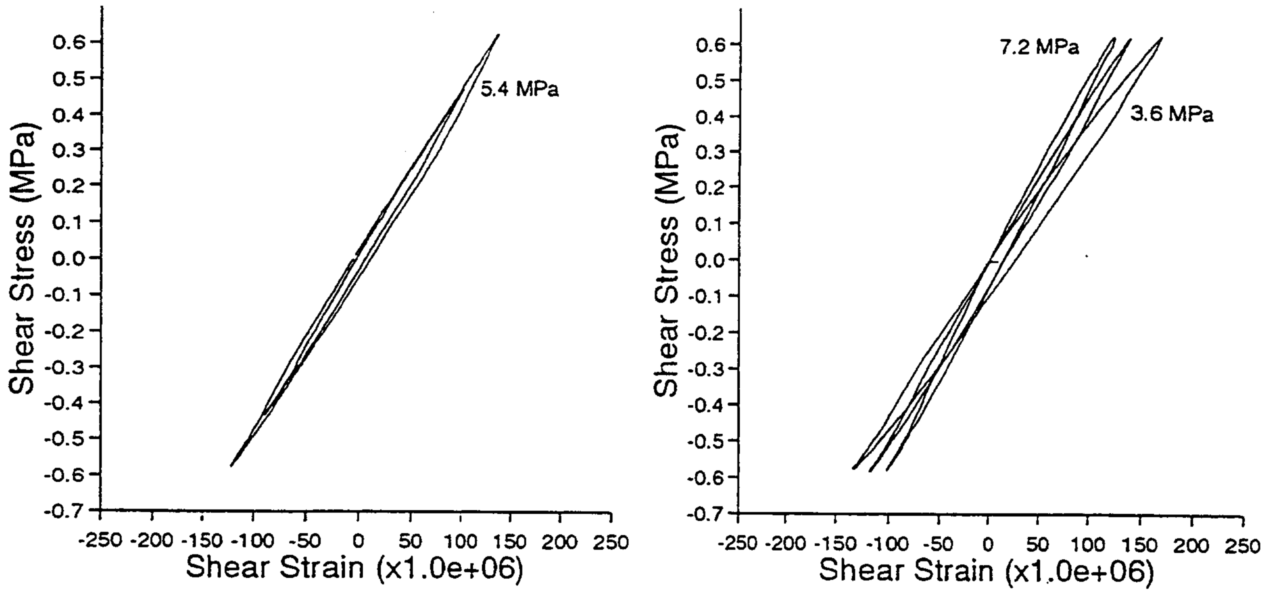


Figure 3: Measured shear stress versus shear strain for Berea sandstone. In the left figure, the effect of shear stress oscillation amplitude is shown at constant axial load (5.4 MPa). Note that the smaller loop is nested within and rotated with respect to the larger loop. In the right figure, the effect of normal load is illustrated, with loops at three different normal loads (3.6, 5.4, and 7.2 MPa). Note that the shear modulus increase and the loop width decreases with increasing normal load.

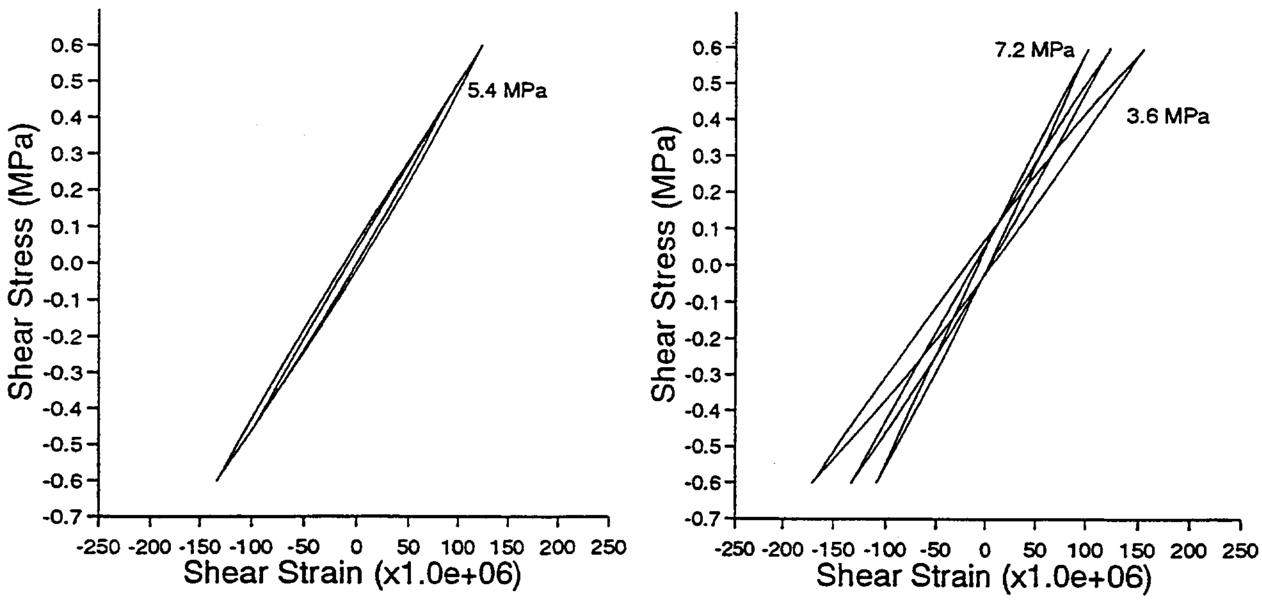


Figure 4: Predicted shear stress versus shear strain for Berea sandstone experiments in Figure 3. A simple "bow-tie" rheology was used to model the data. The model does a good job in producing both the amplitude and axial stress dependence of the deformation.

As we might expect, the hysteresis loops appear to exhibit the same "discrete memory" and "closed-loop" characteristics as noted in studies of hysteresis in Youngs modulus during uniaxial-stress perturbations. Based on this preliminary data, the hysteresis in the shear modulus appears relatively easy to model with only a few parameters. As an example, a simple "bow-tie" model for the shear modulus as a function of shear stress represents the observations fairly well. In the "bow-tie" example, the shear modulus is assumed to follow the form $G = G' - A \cdot |\tau - \tau_r|$, where G' is a function of the axial load (σ_{11}), τ is the shear stress, and τ_r is the shear stress at the last reversal in loading direction. In Figure 4 we plot the predicted stress-strain behavior for the experimental cases shown in Figures 3a and 3b. In Figures 4a and 4b, all the data was modeled using $G' = 2520 + 555\sigma_{11}$ (where G' and σ_{11} are in MPa), and $A = 0.001$ is assumed constant.

Confined Compression and Damage

Preliminary experiments have also been conducted on Barre granite as exploratory experiments for studying the effect of confining pressure on uniaxial stress deformation and for extending the moderate strain regime rheology into the brittle damage regime. Figure 5 illustrates the results of two similar experiments performed at two different confining pressures (0 MPa and 10 MPa respectively).

The experiments shown in Figure 5 consisted of a gradually increasing differential load at fixed confining pressure. During the loading, numerous small perturbations in the load were imposed to measure the unloading stiffness. In these experiments we find that once we enter the damage regime (i.e. differential stress levels above 150 MPa), the stress-strain relationship during cyclic loading exhibits a ratcheting effect, which becomes more pronounced as we approach failure. This ratcheting is thought to reflect time dependent crack growth.

The data is currently being analyzed to help constrain a damage-mechanics based model of rock rheology for the brittle-damage regime (see *Sammis* this issue). In addition the data also provides some preliminary constraint on confining pressure effects in the moderate-strain regime. In Figure 5b, the logarithm of the local Youngs modulus (E) during loading for each experiment is plotted using solid lines and the logarithm of the unloading stiffnesses (E') are plotted using circles. Looking at the data in the moderate-strain regime (differential stresses below 150 MPa), we see that the effect of confining pressure on both E and E' can be modeled with a simple scale factor which is of comparable magnitude for both E and E'. This is consistent with the γ parameter as discussed earlier, although the data indicates that γ is a function of differential stress at low axial loads (a complication ignored in our earlier treatment). Also note that once in the brittle-damage regime, this simple scaling with confining pressure breaks down, as the effect of confining pressure on E develops a more pronounced pressure effect than on E'.

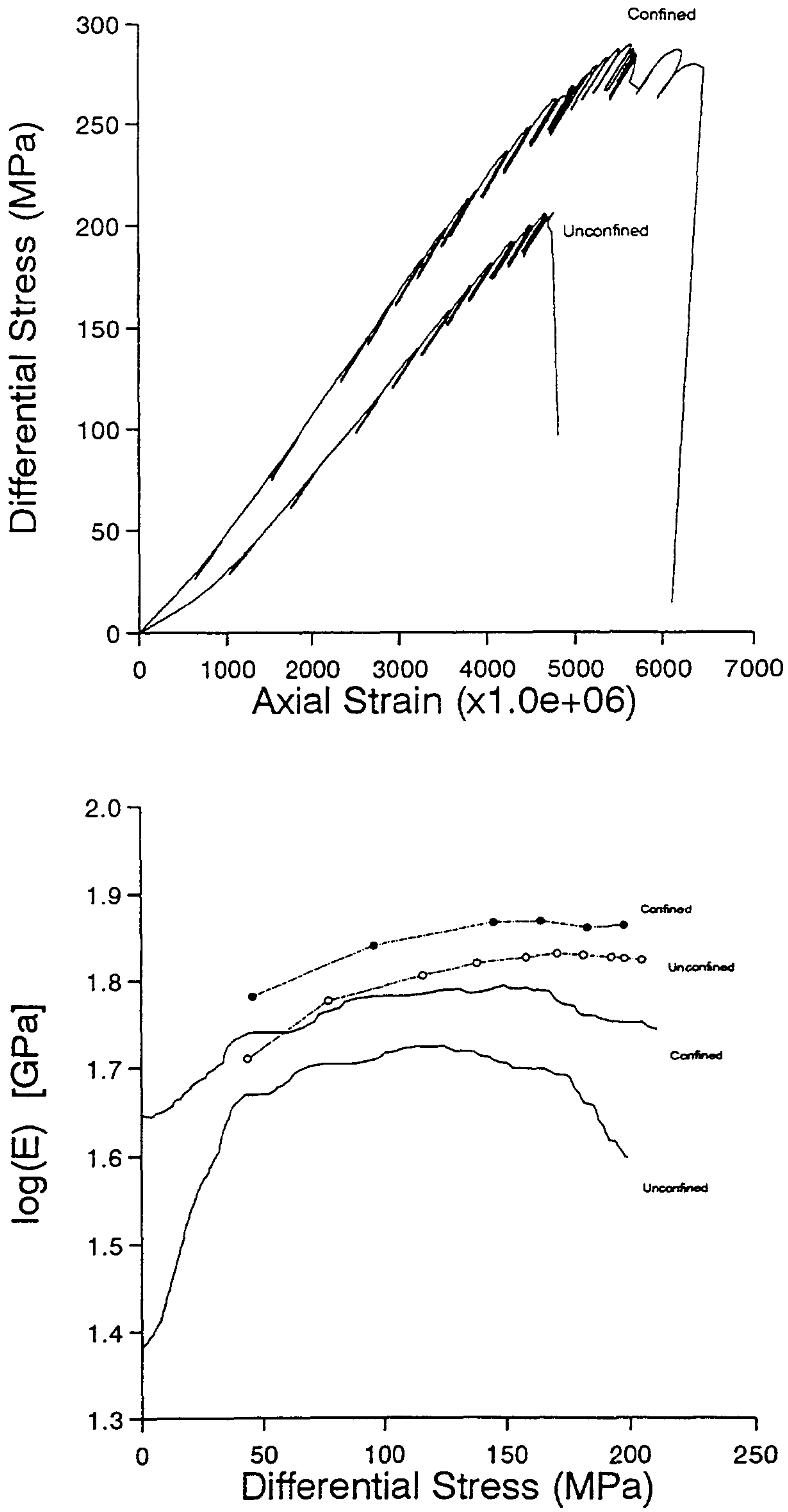


Figure 5: Stress strain data from two uniaxial compression experiments on Barre granite, one unconfined and the other at a fixed confining pressure of 10 MPa. During the loading, perturbations in loading were performed to measure the unloading stiffness. In the lower figure, the logarithm of the Youngs modulus is plotted as a function of differential stress. Solid lines indicating the modulus upon loading and the circles indicate the average modulus during the small cyclic perturbations.

RECOMMENDATIONS AND FUTURE PLANS

The preliminary results shown here are being used to design experiments for constraining rheologies needed to model wave propagation in the moderate-strain and brittle-damage regimes.

Emphasis of future experiments will be placed in a number of important areas. We will improve the quality of torsional data for constraining hysteresis in the shear modulus, so that details of the rheology can be quantified. In addition, based on the preliminary results shown here, we will design and perform experiments to quantify the confining pressure effect on the uniaxial-stress rheology and explicitly test to see if a simple scaling factor (i.e. γ) can be used, along with the uniaxial stress-rheology, to model hysteresis for loading paths transitional between uniaxial-stress and uniaxial strain. The experiments will include testing in the damage regime, and in so doing provide a data set with which to integrate rheologies for the moderate-strain and brittle damage-regimes. We also plan perform additional experiments designed to better quantify the details of the observed cusps in the stress-strain relationship for both shear and uniaxial-stress perturbations. Tests are planned on Berea Sandstone, Sierra White granite, Barre granite, and welded tuff recovered from N-Tunnel at NTS.

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